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DEPENDENCE OF THE CHARACTERISTICS OF RADIATIVE HEAT EXCHANGE ON THE OPTICAL PROPERTIES OF ABSORBING, EMITTING, AND SCATTERING MEDIA AND THEIR BOUNDARY SURFACES

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We suggest a new hybrid numerical-analytical method for solving an integrodifferential equation of radiation transfer. We present results of an investigation of the effect of the temperature distribution and scattering properties of a medium and its absorption density as well as the emissivity of the boundary on the intensity and density of the emergent-radiation fluxes.

Radiative heat transfer plays an exceptionally important part in modern engineering and technology. In the furnace chambers of waste-heat boilers and steam generators, metallurgical furnaces, high-temperature reactors of chemical technology, and many other facilities operating at high temperatures radiation is the basic mode of energy transfer.

In combustion chambers of power plants the heat carrier is usually a two-phase system of gas–solid (or liquid) particles. The presence of condensed-phase particles of the combustion products is responsible for both their additional contribution to the overall radiation of the heat carrier and the occurrence of multiple processes of scattering that influence the angular distribution of the radiation in the medium. Among the main gaseous components of the combustion products that are optically active in the infrared spectral region there are CO_2, H_2O, and CO. Particles of soot, ash, and coke constitute the condensed phase of the heat carrier.

The optical properties of the medium and its boundary surface affect appreciably the intensity of radiative heat exchange. The ever-increasing requirements on the reliability, economic efficiency, and environmental safety of high-temperature technological facilities necessitate a more detailed investigation of this phenomenon and allowance for it.

Below we present investigation results for the dependence of the intensity and density of the radiation fluxes incident on the boundary surfaces of a medium on the temperature distribution, its scattering properties and absorption density, and the emissivity of the boundary.

To eliminate effects that may appear due to the shape of the volume occupied by the medium, the investigations were carried out for a plane layer whose thickness was taken equal to unity and were based on numerical solution of an integrodifferential equation for radiation transfer that was studied assuming local thermodynamic equilibrium [1]:

$$\mu \frac{\partial}{\partial x} I(x, \mu) + (\chi(x) + \sigma(x)) I(x, \mu) =$$

$$= \chi(x) B(T(x)) + \frac{\sigma(x)}{2} \int_{-1}^{1} p(x, \mu, \mu') I(x, \mu') d\mu' , \quad (1)$$

$$0 \le x \le 1 , \quad -1 \le \mu \le 1 .$$

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Here $I(x, \mu)$ is the intensity of the radiation at the point with the coordinate x in the direction forming the angle $\Theta = \arccos \mu$ with the coordinate axis; $\chi(x)$ and $\sigma(x)$ are the coefficients of absorption and scattering, respectively; $p(x, \mu, \mu')$ is the radiation scattering phase function for an elementary volume of the medium; $B(T)$ is the intensity of the radiation of a blackbody at the temperature T.

The scattering phase function $p(x, \mu, \mu')$ is determined by the fraction of the energy coming to the point x from the direction μ' and scattering in the direction μ. In [2, 3] it was shown that in a number of cases (for example, when the radiation is multiply scattered on particles of soot, ash, or coke) the scattering phase function can be represented in the form

$$p(x, \mu, \mu') = \beta(x) + 2[1 - \beta(x)]\delta(\mu - \mu'), \tag{2}$$

where $\beta(x)$ is the doubled fraction of backward scattering of the radiation upon its interaction with an elementary volume of the medium, and $\delta(\mu - \mu')$ is the delta-function.

With allowance for Eq. (2) the problem is reduced to the case of isotropic scattering, and Eq. (1) takes the form

$$\mu \frac{\partial}{\partial x} I(x, \mu) + \alpha(x) I(x, \mu) = S(x, I), \tag{3}$$

where $S(x, I) = \chi(x)B(x) + \sigma'(x)/2 \int_{-1}^{1} I(x, \mu')d\mu'$ is the function of sources, and $\alpha(x) = \chi(x) + \sigma(x)$, $\sigma' = \beta\sigma$.

In carrying out these investigations, we considered different kinds of boundary conditions. Thus, for example, when we investigated the dependence of the intensity and density of the fluxes of emergent radiation, on the optical density and scattering properties of the medium, we assumed that the boundary surfaces were transparent in order to exclude the influence of their optical properties:

$$I(0, \mu)|_{\mu>0} = I_1(\mu) \quad \text{and} \quad I(1, \mu)|_{\mu<0} = I_2(\mu). \tag{4}$$

Here I_1 and I_2 are the intensity of the radiation incident from the outside onto the left and right boundaries of the layer considered, respectively.

When we investigated the effect of the emissivity of the boundary surfaces on the characteristics of the radiative heat exchange, we assumed the boundaries of the layer to be diffusely reflecting and emitting:

$$I(0, \mu)|_{\mu>0} = \varepsilon B(T_w) + 2(1 - \varepsilon) \int_{-1}^{0} I(0, \mu')\mu' d\mu',$$

$$I(1, \mu)|_{\mu>0} = I(0, -\mu). \tag{5}$$

In a number of cases the use of the method of finite differences for a numerical solution of Eq. (3) gave results that did not have a physical meaning. To overcome this difficulty, the present authors suggested a new hybrid numerical-analytical method that turned out to be an extremely powerful technique for numerical solution of radiation problems. The essence of it is as follows.

We consider the following formal solution of Eq. (3) with boundary conditions (4):

$$I(x, \mu) = \exp\left(-\int_0^x \frac{\alpha(t)}{\mu} dt\right) \left(I_1(\mu) + \int_0^x \frac{S(t, I)}{\mu} \exp\left(\int_0^t \frac{\alpha(t')}{\mu} dt'\right) dt\right), \quad \mu > 0;$$

$$I(x, \mu) = \exp\left(-\int_1^x \frac{\alpha(t)}{\mu} dt\right) \left(I_2(\mu) + \int_1^x \frac{S(t, I)}{\mu} \exp\left(\int_1^t \frac{\alpha(t')}{\mu} dt'\right) dt\right), \quad \mu < 0; \tag{6}$$

$$I(x,\mu) = \frac{S(x, I)}{\alpha(x)}, \quad \mu = 0 .$$

If the quantities α and S are independent of the coordinate, then we can simplify expressions (6) substantially:

$$\begin{aligned} \mu > 0: \quad & I(x,\mu) = \exp\left(-\frac{\alpha x}{\mu}\right)\left(I_1(\mu) - \frac{S}{\alpha}\right) + \frac{S}{\alpha}, \\ \mu < 0: \quad & I(x,\mu) = \exp\left(-\frac{\alpha(x-1)}{\mu}\right)\left(I_2(\mu) - \frac{S}{\alpha}\right) + \frac{S}{\alpha}. \end{aligned} \tag{7}$$

The proposed method of solution is based on piecewise-analytical solutions (7) and consists in combining them with the method of discrete ordinates and subsequent iterative refinement of the function of sources.

Following the method of discrete ordinates, we select N_Θ directions of propagation of radiation along which we calculate the radiation field ($0 \le \Theta \le \pi$). For each selected direction Eq. (3) is written in the form

$$\mu^k \frac{\partial}{\partial x} I^k(x) + \alpha(x) I^k(x) = S(x, I), \quad 1 \le k \le N_\Theta, \tag{8}$$

where $\mu^k = \cos\Theta_k$; $I^k(x)$ is the intensity of the radiation at the point x in the k-th direction.

Next, we divide the layer into sublayers sufficient in number that the values of the functions α and S change little within each layer. As a result, we obtain a computational grid along the coordinate x with a certain number of nodes N_{calc}. It is assumed that within the limits of the i-th sublayer $\alpha = (\alpha_i + \alpha_{i+1})/2$ and $S = (S_i + S_{i+1})/2$. Then, taking account of Eq. (7), it is possible to obtain recursion formulas for determining the radiation intensity at all the nodes of the computational grid $1 \le i \le N_{\text{calc}}$ for all the selected directions of radiation transfer $1 \le k \le N_\Theta$:

$$\begin{aligned} \underline{\mu^k > 0}: \quad & I_{i+1}^k = \exp\left(-\frac{\alpha \Delta x}{\mu^k}\right)\left(I_i^k - \frac{S}{\alpha}\right) + \frac{S}{\alpha}, \\ \underline{\mu^k < 0}: \quad & I_i^k = \exp\left(\frac{\alpha \Delta x}{\mu^k}\right)\left(I_{i+1}^k - \frac{S}{\alpha}\right) + \frac{S}{\alpha}, \end{aligned} \tag{9}$$

$$\Delta x = x_{i+1} - x_i .$$

Using formulas (9) and boundary conditions (4)

$$I(0, \mu^k)|_{\mu^k > 0} = I_1^k = I_1(\mu^k) \quad \text{and} \quad I(1, \mu^k)|_{\mu^k < 0} = I_{N_{\text{calc}}}^k = I_2(\mu^k),$$

we can calculate the radiation field at all points of the computational region. However, due to the dependence of the function of sources $S(x, I)$ on the radiation intensity, iterations are required for its refinement. In this case, to calculate the integral term we can, for example, use the trapezoidal formula

$$\int_{-1}^{1} I(x_i, \mu')\, d\mu' = 0.5 \sum_{k=1}^{N_\Theta - 1} (I_i^k + I_i^{k+1})\, |\mu^{k+1} - \mu^k| . \tag{10}$$

Thus, the entire calculation process can be divided into the following stages:

1) the values of the radiation intensity I_i^k are assumed to be equal to zero for all of the nodes of the computational grid ($1 \le i \le N_{\text{calc}}$) and for all of the directions considered ($1 \le k \le N_\Theta$);

2) the values of the function of sources S_i ($1 \le i \le N_{\text{calc}}$) are calculated according to formula (10);

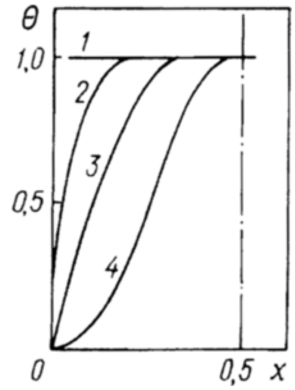


Fig. 1. Distributions of the dimensionless temperature over the coordinate x.

3) for each of the directions ($1 \le k \le N_\Theta$) we calculate the value of the intensity of the radiation I_i^k using formula (9). Simultaneously we determine the value of the relative discrepancy of neighboring iterations $\delta = \max_i |1 - I_i^{k,m}/I_i^{k,m+1}|$, m is the iteration numbers;

4) we check the condition $\delta \le \delta_0$ (δ_0 is the given discrepancy); if this condition is not satisfied, the calculation is repeated beginning from Item 2.

The above computational process makes it possible to calculate with any prescribed accuracy the radiation intensity in any given direction at each point of a plane layer of an absorbing, emitting, and scattering medium. Numerical experiments showed that the iteration process converges in 3–10 iterations depending on the kind of boundary conditions and the Schuster number.

Thus, we consider a plane layer of a selectively emitting, absorbing, and scattering medium. We analyze the results of a numerical investigation of the effect of its absorption density and scattering properties on the intensity and density of the emergent radiation. In this case the absorption density of the medium is determined by the integral $\tau = \int_0^1 \chi(x)dx$, and its scattering properties are characterized by the Schuster number $\mathrm{Sc} = \int_0^1 \sigma(x)/\chi(x))dx$.

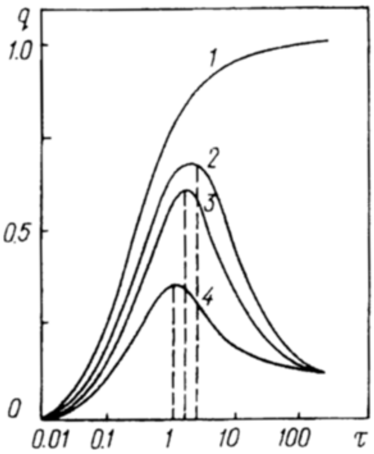
The density of the radiation fluxes emerging from a layer of unit optical thickness is determined as

$$Q(0) = 2\pi \int_{-1}^{0} I(0, \mu')\mu' \, d\mu' ; \quad Q(1) = 2\pi \int_{0}^{-1} I(1, \mu')\mu' \, d\mu' .$$

For greater clarity of graphical information, in what follows we use the reduced values of these quantities defined as their ratios to the Planck radiation density at the maximum temperature of the medium $T_{\max}$: $q(0) = Q(0)/(\pi B(T_{\max}))$ and $q(1) = Q(1)/(\pi B(T_{\max}))$. Similarly we normalize the intensity of the radiation emerging from the layer: $I_w = I/B(T_{\max})$.

We carried out investigations for temperature distributions typical of the furnace chambers of power plants. It is known that in the most cases a symmetric temperature profile is formed in combustion chambers. Moreover, at a certain distance from the level of the burners along the height of the furnace the temperature of the gases at the center of the flame is equalized [4], so that a constant-temperature core is formed at the center. As the number of burners increases, the probability of the occurrence of a constant-temperature core in the furnace volume increases. However, farther along the height of the flame, the temperature profile in the cross section of the furnace changes under the influence of convection and approaches a Schlichting-type profile [5] that characterizes the temperature distribution in a developed turbulent flow. Figure 1 presents the considered distributions of the dimensionless temperature over the coordinate x:

$$\Theta(x) = (T(x) - T_b)/(T_{\max} - T_b) ,$$



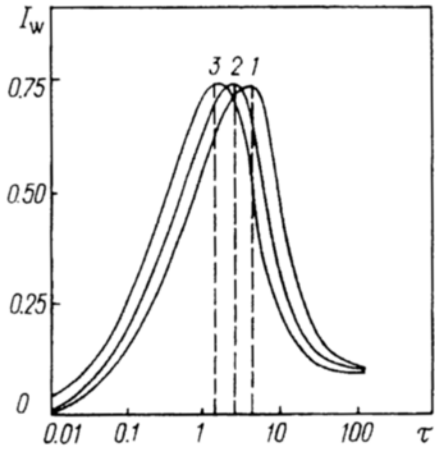


Fig. 2. Emergent-radiation flux density vs. absorption density of the medium. The numbers of the curves correspond to the temperature distributions in Fig. 1.

Fig. 3. Effect of the optical density of the medium on the intensity of the emergent radiation propagating in the direction $\Theta = 0$ (1), $\pi/4$ (2), $3\pi/8$ (3).

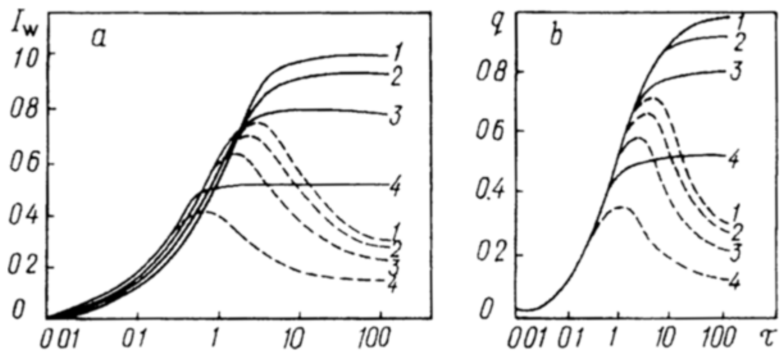


Fig. 4. Effect of scattering on the intensity (a) and density (b) of fluxes of emergent radiation in a homogeneous (solid lines) and inhomogeneous (dashed lines) layer for Sc = 0 (1), 0.3 (2), 0.6 (3), 0.9 (4).

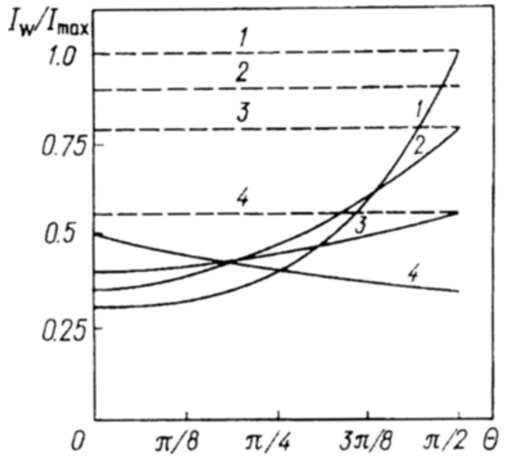
where T_b is the temperature of the medium at the layer boundary. The temperature of the boundary surface T_w, as a rule, differs from T_b in actual furnaces. Usually, $T_w \le T_b$ due to the radiation slip effect.

The values of the temperature and optical density of the medium and the Schuster number and emissivity of the boundary surface are varied within the ranges characteristic for the furnace chambers of power plants. All the results that are discussed below were obtained on the assumption of a uniform distribution of the radiation absorption and scattering coefficients over the volume. The emissivity of the heat-absorbing surface was a prescribed constant.

Let us analyze the effect of the optical density of the medium and the temperature distribution in it on the intensity and density of the emergent-radiation fluxes. Here the medium considered is assumed to be nonscattering, so that Sc = 0. External radiation is absent. Results that characterize this effect were obtained for $T_{max} = 2000$ K, $T_{min} = 1000$ K, $\lambda = 3$ μm (Figs. 2 and 3).

An analysis of the data presented in Fig. 2 shows that in the case of a uniform temperature (Fig. 1, curve 1) the values of the emergent-radiation flux density increase monotonical as τ increases, and as $\tau \to \infty$ (actually, for $\tau \ge 100$), $q(\tau) \to 1$.

In a nonhomogeneous medium, for the temperature distributions presented in Fig. 1 (curves 2-4) the value of $q(\tau)$ changes from $I_0/B(T_{max})$ at $\tau = 0$ (I_0 is the intensity of the external radiation, if the latter exists) to



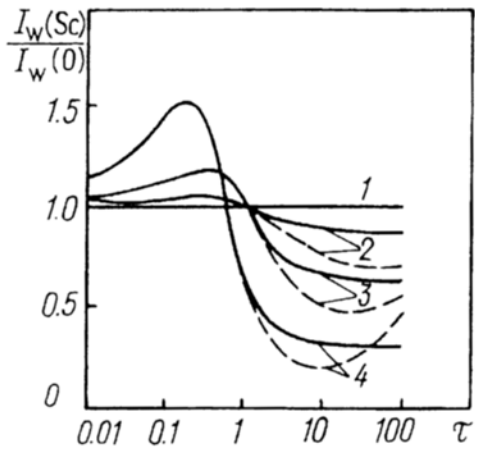


Fig. 5. Distribution of the intensity of radiation emerging from a homogeneous layer over the angle Θ for $\tau = 0.35$ (solid lines) and $\tau = 100$ (dashed lines) and Sc = 0 (1), 0.3 (2), 0.6 (3), 0.9 (4).

Fig. 6. Dependence of the relative value of the emergent-radiation intensity on τ in a homogeneous (solid lines) and inhomogeneous (dashed lines) layer for Sc = 0 (1), 0.3 (2), 0.6 (3), 0.9 (4).

$B(T_{min})/B(T_{max})$ for $\tau \to \infty$ and always has a pronounced maximum at a certain value of τ, which will be called critical in what follows and will be denoted by τ^*. In the temperature range typical for furnace chambers (1000–2000 K) the critical value τ^* virtually depends not on the radiation wavelength or the actual values of the temperature of the medium, but only on the form of its distribution in the layer. It can be calculated from the approximate formula

$$\tau^* = 1/\sqrt[4]{1 - s}\,, \quad s = \int_0^1 \Theta(x)\, dx\,, \tag{11}$$

obtained by processing results of numerical investigations. As s increases, the critical value of the optical density of the medium increases. The corresponding maximum value of the radiation flux density also increases.

The dependence of the intensity of the radiation emerging from the layer I_w on τ in the case of a nonuniform temperature distribution always has a maximum. The critical value τ^* at which the maximum radiation intensity is attained depends greatly on the direction of radiation propagation and decreases with increase in the angle Θ. Here the maximum intensity hardly changes, as is clearly seen from Fig. 3, which illustrates this dependence for the temperature distribution corresponding to curve 3 in Fig. 1.

We also investigated the effect of scattering processes on radiation transfer in a homogeneous absorbing, emitting, and scattering layer with nonirradiated boundaries

$$I(0, \mu)|_{\mu>0} = I_1(\mu) = 0 \quad \text{and} \quad I(1, \mu)|_{\mu<0} = I_2(\mu) = 0\,.$$

Typical dependences of the intensity and density of the radiation fluxes emerging from the medium on its absorption density for different values of the Schuster number are presented in Fig. 4.

In the case of a nonuniform temperature distribution in an absorbing, emitting, and scattering layer with nonirradiated boundaries the functions $I_w(\tau)$ and $q(\tau)$ always have an extremal character (Fig. 4), i.e., there is a critical value of the absorption density of the medium τ^* at which the values of the characteristics of radiation transfer attain their maximum values. As the fraction of the scattering processes increases, the position of the maximum of the values of I_w and q shifts toward smaller values of the optical thickness.

An analysis of results of numerical experiments shows that in the cases considered the radiation scattering processes may lead to both amplification of the emergent-radiation intensity and its attenuation, depending on the

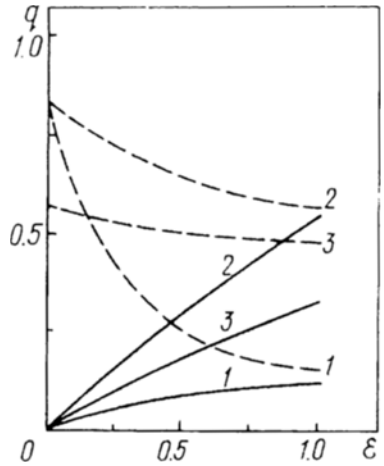


Fig. 7. Dependence of the density of the net flux of radiation (solid lines) and the incident flux of radiation on the boundary surface of the layer (dashed lines) on the surface emissivity for $\tau = 0.1$ (1), 1.0 (2), 10.0 (3).

directed optical thickness of the layer $\tau(\Theta) = \tau/\cos\Theta$ and the Schuster number. This is vividly demonstrated by the angular distribution of the emergent-radiation intensity presented in Fig. 5.

In fact, on the one hand, an increase in the coefficient of total attenuation of the medium $\alpha = \chi + \sigma$ favors attenuation of radiation, and, on the other hand, scattering of radiation leads to an increase in the optical path and, consequently, to amplification of the emergent-radiation intensity due to the self-emission by the medium. The interaction of these two mechanisms can cause both an increase and a decrease in the emergent-radiation intensity, depending on the absorption density of the medium and its scattering properties. It should be noted that in the case considered the amplification and attenuation of the radiation intensity can be substantial and can attain 50% or more, compared to a nonscattering medium (Fig. 6). The fact should also be emphasized that amplification of radiation intensity due to scattering processes is observed only at small optical thicknesses $\tau \le 1$.

An analysis of numerical results shows that in the case considered we may isolate three ranges of values of the optical thickness of the layer that differ by the character of the effect of scattering on the emergent-radiation intensity:

1) when $\tau < 0.5$, the function $I_w(\mathrm{Sc})$ increases;

2) in the range $0.5 \le \tau \le 2$ the function $I_w(\mathrm{Sc})$ has an extremal character;

3) when $\tau > 2$, the function $I_w(\mathrm{Sc})$ decreases.

In spite of the fact that in the first range scattering increases the emergent-radiation intensity, it decreases the greatest possible values of the transfer characteristics (I_w, q) compared to the nonscattering case. We note that a decrease in the indicated values also occurs upon contraction of the "hot" zone.

The effect of the emissivity of the boundary surface on the characteristics of radiative heat exchange was investigated on the example of a plane layer with diffusely emitting and reflecting surfaces (boundary conditions (5)). Results of numerical experiments that characterize the dependence of the density of the net radiation flux and the radiation flux incident on the boundary surfaces of the layer on their optical properties are illustrated in Fig. 7. The curves presented in this figure were obtained for the temperature distribution given by curve 3 in Fig. 1.

In conclusion we mention that the extremal character of the function $q(\tau)$ was already noted in the work of Viskanta [6] when he investigated radiative-convective heat transfer in a nonscattering plane layer. The effect of the temperature profile on the characteristics of radiative heat exchange was considered in [7], and in [8] the existence of a profile of temperatures of a selective gas is indicated at which the radiative flux to the heating surface is maximum.

NOTATION

$I(x, \mu)$, radiation intensity at the point x in the direction $\Theta = \arccos \mu$; Θ, angle between the direction of radiation and the x axis; I_0, intensity of external radiation; I_w, emergent-radiation intensity; q, density of the radiation flux incident on the boundary surface; T_w, T_b, temperature of the boundary surface and temperature of the medium in the direct vicinity of it; $\delta(\mu - \mu')$, delta-function; ε, emissivity of the boundary surface.

REFERENCES

1. K. S. Adzerikho, E. F. Nogotov, and V. P. Trofimov, Radiative Heat Exchange in Two-Phase Media [in Russian], Minsk (1987).
2. K. S. Adzerikho and V. P. Nekrasov, Inzh.-Fiz. Zh., **34**, No. 5, 894-896 (1978).
3. K. S. Adzerikho and V. P. Nekrasov, Inzh.-Fiz. Zh., **22**, No. 1, 168-169 (1972), Deposited at VINITI, No. 2834-71.
4. S. A. Shagalova and I. N. Shnittser, Burning of a Solid Fuel in Furnaces of Steam Generators [in Russian], Leningrad (1976).
5. N. V. Kuznetsov et al. (eds.), Thermal Calculation of Boiler Plants (Standard Method) [in Russian], Leningrad (1976).
6. R. Viskanta, J. Heat Transfer, No. 85, 318-328 (1963).
7. A. A. Ots, Processes in Steam Generators in Burning Shales and Kansk-Achinsk Coals [in Russian], Moscow (1977).
8. V. N. Golovin, Izv. VUZov, Teploénergetika, No. 7, 6-9 (1964).